## Mark Scheme (Results)

January 2024

Pearson Edexcel International Advanced Level in Mechanics M3 (WME03) Paper 01

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January 2024
Question Paper Log Number P74324A
Publications Code WME03_01_2401_MS
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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## General Instructions for Marking

The total number of marks for the paper is 75 .
Edexcel Mathematics mark schemes use the following types of marks:

## 'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation, e.g. resolving in a particular direction; taking moments about a point; applying a suvat equation; applying the conservation of momentum principle; etc.

The following criteria are usually applied to the equation.
To earn the M mark, the equation
(i) should have the correct number of terms
(ii) each term needs to be dimensionally correct

For example, in a moments equation, every term must be a 'force x distance' term or 'mass $x$ distance', if we allow them to cancel ' $g$ ' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.
' M ' marks are sometimes dependent (DM) on previous $M$ marks having been earned, e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity - this M mark is often dependent on the two previous $M$ marks having been earned.
'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. MO A1 is impossible.
'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).
$A$ and $B$ marks may be f.t. - follow through - marks.

## General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod means benefit of doubt
- ft means follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao means correct answer only
- cso means correct solution only, i.e. there must be no errors in this part of the question to obtain this mark
- isw means ignore subsequent working
- awrt means answers which round to
- SC means special case
- oe means or equivalent (and appropriate)
- dep means dependent
- indep means independent
- dp means decimal places
- sf means significant figures
-     * means the answer is printed on the question paper
- $\square$ means the second mark is dependent on gaining the first mark

All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

(NB specific mark schemes may sometimes override these general principles)

- Rules for M marks:
- correct no. of terms;
- dimensionally correct;
- all terms that need resolving (i.e. multiplied by cos or $\sin$ ) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark, i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g=9.8$ should be given to 2 or 3 SF .
- Use of $\mathrm{g}=9.81$ should be penalised once per (complete) question.
- N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c)...then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads - if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft


## Mechanics Abbreviations

M(A) Taking moments about A.
N2L Newton's Second Law (Equation of Motion)
NEL Newton's Experimental Law (Newton's Law of Impact)
HL Hooke's Law
SHM Simple harmonic motion
PCLM Principle of conservation of linear momentum
RHS Right hand side
LHS Left hand side

| 1a | $-\frac{m g R^{2}}{2 x^{2}}=m v \frac{\mathrm{~d} v}{\mathrm{~d} x}$ | M1 | Form differential equation in $v$ and $x$ only. Need to see $\frac{\mathrm{d} v}{\mathrm{dx}}$ or $v \frac{\mathrm{~d} v}{\mathrm{~d} x}$ Cannot get this mark using $t$. Allow with both $m$ 's cancelled. Condone sign error. |
| :---: | :---: | :---: | :---: |
|  | $-\frac{g R^{2}}{2} \int \frac{1}{x^{2}} \mathrm{~d} x=\int \nu \mathrm{d} v$ | M1 | Separate variables correctly and integrate at least one side. Cannot get this mark using $t$. Condone sign error. |
|  | $v^{2}=\frac{g R^{2}}{x}+C *$ | A1* | Obtain given answer from correct work. Must include at least one line of working between integral and final answer. Correct signs seen throughout working. <br> Condone $\frac{v^{2}}{2}=\frac{g R^{2}}{2 x}+C$ followed by $v^{2}=\frac{g R^{2}}{x}+C$ <br> Note: If the first line of working is $\frac{1}{2} v^{2}=-\int \frac{g R^{2}}{2 x^{2}} d x$ followed by integration of RHS, this scores M0M1A0* |
| ALT1 <br> (a) | $\frac{1}{2} m u^{2}-\frac{1}{2} m v^{2}=\int \frac{g m R^{2}}{x^{2}} \mathrm{dx}$ | M1 | Form an energy equation with 2 KE terms and the integral of the variable force. Condone sign errors. |
|  | $\frac{1}{2} m u^{2}-\frac{1}{2} m v^{2}=-\frac{g m R^{2}}{x}+A$ | M1 | Integrate the force wrt $x$. Condone sign errors. |
|  | $v^{2}=\frac{g R^{2}}{x}+C \quad *$ | A1* | Obtain given answer from correct work. Must include at least one line of working and correct signs seen throughout working. |
|  |  | [3] |  |
| b | $x=3 R, v^{2}=3 g R$ | M1 | Use initial conditions to evaluate $C$ in the given answer. |
|  | $\Rightarrow C=3 g R-\frac{g R^{2}}{3 R}\left(=\frac{8 g R}{3}\right)$ | A1 | Or equivalent |
|  | $x=R \Rightarrow v=\sqrt{\frac{11 g R}{3}}$ | A1 | Accept $\frac{\sqrt{33 g R}}{3}$ <br> Answer must be in terms of $g$ and $R$ |
|  |  | [3] |  |
| $\begin{array}{\|c\|} \hline \text { ALT1 } \\ \text { (b) } \\ \hline \end{array}$ | Use of definite integral instead of finding $+C$ |  |  |
|  | $\left[v^{2}\right]_{\sqrt{38 R}}^{v}=\left[\frac{g R^{2}}{x}\right]_{3 R}^{R}$ | M1 | Use initial conditions in a definite integral. |


|  | $v^{2}-3 g R=\frac{g R^{2}}{R}-\frac{g R^{2}}{3 R}$ | A1 | Or equivalent |
| :--- | :---: | :---: | :--- |
| $v=\sqrt{\frac{11 g R}{3}}$ | A1 | Accept $\frac{\sqrt{33 g R}}{3}$ <br> Answer must be in terms of $g$ and $R$ |  |
|  |  | $(6)$ |  |


| 2a | $\begin{aligned} & \text { Change in GPE } \\ & =m g \times 1.3 l \sin \theta(=0.5 l m g) \end{aligned}$ | M1 | Condone sin / cos confusion |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{EPE}=\frac{\lambda l^{2}}{2 l} \text { or } \mathrm{EPE}=\frac{\lambda(0.3 l)^{2}}{2 l}$ | B1 | One correct term for EPE |
|  | Energy equation $B$ to $A$ | M1 | Dimensionally correct with all the required terms. Condone sign errors and $\sin / \cos$ confusion |
|  | $\frac{\lambda l^{2}}{2 l}-\frac{\lambda(0.3 l)^{2}}{2 l}=0.5 \mathrm{lmg}$ | A1 | Correct unsimplified equation |
|  | $\Rightarrow \lambda=m g \frac{1}{1-0.09}=\frac{100}{91} m g$ | A1* | Obtain given answer from correct working. Must see evidence of simplification. |
|  |  | [5] |  |
| 2b | Equation of motion | M1 | Dimensionally correct with all the required terms. Condone sign errors and $\sin / \cos$ confusion. |
|  | $T-m g \sin \theta=m a$ | A1 | Correct unsimplified equation |
|  | $\begin{aligned} & \frac{\lambda \times l}{l}-m g \sin \theta=m a \\ & \left(\frac{100}{91} m g-\frac{5}{13} m g=m a\right) \end{aligned}$ | A1 | Correct unsimplified equation with HL used to replace $T$ |
|  | $a=\frac{5}{7} g$ | A1 | Accept 0.71 g or better. If $g=9.8$ is used, accept 7 . |
| Note: If $g=9.81$ is used then penalise once per complete question. SHM equations can only be used if the motion is proven to be SHM first. | Note: If $g=9.81$ is used then penalise once per complete question. SHM equations can only be used if the motion is proven to be SHM first. |  |  |
|  |  | [4] |  |
|  |  | (9) |  |


| 3a | Moment of $S$ about the $y$-axis | M1 | Use of formula $(\pi)(\rho) \int x y^{2} \mathrm{~d} x$ <br> No need to see the correct limits here. The curve equation must be substituted correctly and an attempt to integrate seen (at least one term must have a power of $x$ raised by 1 ) Note the correct expression for integrating is $\begin{aligned} & x\left(\frac{1}{4} x(3-x)\right)^{2}=\frac{1}{16} x^{3}(3-x)^{2} \\ & =\frac{1}{16}\left(9 x^{3}-6 x^{4}+x^{5}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
|  | $=(\pi)(\rho) \frac{1}{16}\left[\frac{9}{4} x^{4}-\frac{6}{5} x^{5}+\frac{1}{6} x^{6}\right]$ | A1 | Correct integrated expression. |
|  | $=\frac{31}{60}(\pi)(\rho)$ | A1 | Correct use of correct limits ( 0 and 2). No need to see a line of working showing substitution of limits. <br> However, must see $\frac{31}{60}$ or equivalent numerical evaluation of integral. |
|  | $\bar{x}=\frac{\frac{31}{60}(\pi)(\rho)}{\frac{2}{5}(\pi)(\rho)}$ | M1 | Complete method to find the distance. Formula must be the right way up $\bar{x}=\frac{(\pi)(\rho) \int x y^{2} \mathrm{~d} x}{M}$ Must have consistent use of $\pi$ and of $\rho$. |
|  | $=\frac{31}{24} *$ | A1* | Obtain given answer from correct working |
|  |  | [5] |  |
| 3b | Correct use of trig | M1 | Correct trig ratio to find a relevant angle, $\alpha^{\circ}$ or $(90-\alpha)^{\circ}$ <br> Must use curve equation with $x=2$ and $\left(2-\frac{31}{24}\right)$ |
|  | $\tan \alpha^{\circ}=\frac{1}{2} \div \frac{17}{24}\left(=\frac{12}{17}\right)$ | A1 | Or equivalent. Condone reciprocal. |
|  | $\alpha=35$ | A1 | 2 sf or better (35.2175...) A0 for use of radians. |
|  |  | [3] |  |
|  |  | (8) |  |


| 4 |  |  | If angle is between incline and vertical then $\sin \theta=\frac{4}{5}, \cos \theta=\frac{3}{5}$ |
| :---: | :---: | :---: | :---: |
|  | Resolve vertically | M1 | Need all terms. Dimensionally correct. Condone sign errors and $\sin / \cos$ confusion. |
|  | $R \sin \theta=m g+F \cos \theta$ | $\begin{array}{\|l} \hline \mathrm{A} 1 \\ \mathrm{~A} 1 \\ \hline \end{array}$ | Unsimplified equation with at most one error. <br> Correct unsimplified equation |
|  | Equation for horizontal motion | M1 | Need all terms. Dimensionally correct. Condone sign errors and $\sin / \cos$ confusion. Accept any form of acceleration for the method mark only. |
|  | $R \cos \theta+F \sin \theta=m r \omega^{2}$ | A1 <br> A1 | Unsimplified equation with at most one error. Direction of $F$ consistent with vertical resolution. Incorrect form of acceleration is one error. <br> Correct unsimplified equation |
|  | Use of $F=\mu R$ | M1 | Used, not just quoted. $F=\frac{1}{4} R$ |
|  | Substitute for trig and solve for max $\omega$ | DM1 | Dependent on all preceding M marks. If more than two equations are produced, the correct two must be used. $\left(R=\frac{20 m g}{13}, F=\frac{5 m g}{13}\right)$ |
|  | $\Rightarrow \omega=\sqrt{\frac{16 g}{13 r}}$ | A1* | Obtain given answer from correct working |
|  |  | [9] |  |
|  |  | (9) |  |
| Alt1 | Using N2L parallel and perpendicular to the incline. <br> Perpendicular $R-m g \sin \theta=m r \omega^{2} \cos \theta$ <br> Parallel $F+m g \cos \theta=m r \omega^{2} \sin \theta$ | M1 <br> A1A1 <br> M1 <br> A1A1 | Need all terms. Dimensionally correct. Condone sign errors and $\sin / \cos$ confusion. Note that the acceleration must have a $\sin / \cos$ component. Accept any form of acceleration for the method mark only. Mark A's as above. <br> A1A0 Unsimplified equation with at most one error <br> A1A1 Correct unsimplified equation |


| 5 |  |  | Curve equation $x^{2}+y^{2}=r^{2}$ |
| :---: | :---: | :---: | :---: |
| 5a | Using $x$-axis $(\rho) \int x \times 2 \sqrt{r^{2}-x^{2}} \mathrm{~d} x$ <br> or <br> Using $y$-axis $(\rho) \frac{1}{2} \int 2\left(\sqrt{r^{2}-x^{2}}\right)^{2} \mathrm{~d} x$ | M1 | Use of correct integral. Limits not needed here. <br> Accept an integral of the form: <br> $x$-axis: $k \int x \sqrt{r^{2}-x^{2}} \mathrm{~d} x$ <br> $y$-axis: $k \int r^{2}-x^{2} \mathrm{~d} x$ |
|  | $\begin{aligned} & x \text {-axis } \\ & y \text {-axis }=-\frac{2}{3}(\rho)\left(r^{2}-x^{2}\right)^{\frac{3}{2}} \\ & \quad=(\rho)\left(x r^{2}-\frac{x^{3}}{3}\right) \end{aligned}$ | A1 | Correct integration, ignore limits. Correct expression. |
|  | $=\frac{2}{3}(\rho) r^{3}$ | A1 | Correct use of limits, 0 and $r$ or $-r$ and $r$. |
|  | Using $x$-axis $\frac{1}{2} \pi r^{2} \rho \bar{x}=\rho \int_{0}^{r} 2 x y \mathrm{~d} x$ <br> Using $y$-axis $\frac{1}{2} \pi r^{2} \rho \bar{y}=\rho \frac{1}{2} \int_{-r}^{r} y^{2} \mathrm{dx}$ <br> or $\frac{1}{2} \pi r^{2} \rho \bar{y}=\rho \int_{0}^{r} y^{2} \mathrm{dx}$ | M1 | Complete method to obtain distance. <br> Use of a correct formula, consistent with the axis and limits used, to find centre of mass with curve equation. <br> $\rho$ must appear on both sides or neither. |
|  | $\bar{x}=\frac{\frac{2}{3} r^{3}}{\frac{1}{2} \pi r^{2}}=\frac{4 r}{3 \pi}$ | A1* | Obtain given answer from correct working |
| $\begin{gathered} \hline \text { ALT } \\ 1 \\ 5(\mathrm{a}) \end{gathered}$ | Parametric approach $x=r \cos \theta, y=r \sin \theta$ |  | Curve equation $x^{2}+y^{2}=r^{2}$ |
|  | Using $x$-axis $2 r^{3} \int_{0}^{\frac{\pi}{2}} \sin ^{2} \theta \cos \theta \mathrm{~d} \theta$ | M1 | Use of correct integral. Limits not needed here. <br> Accept an integral of the form: $k r^{3} \int \sin ^{2} \theta \cos \theta \mathrm{~d} \theta$ |


|  | $=2 r^{3}\left[\frac{\sin ^{3} \theta}{3}\right]_{0}^{\frac{\pi}{2}}$ |  |  |  | A1 | Correct integration, ignore limits. Correct expression. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $=\frac{2}{3} r^{3}$ |  |  |  | A1 | Correct use of limits |
|  | $\bar{x}=\frac{\frac{2}{3} r^{3}}{\frac{1}{2} \pi r^{2}}$ |  |  |  | M1 | Complete method to obtain distance. <br> Use of correct formula. $\rho$ must appear on both sides or neither. |
|  | $=\frac{4 r}{3 \pi} *$ |  |  |  | A1* | Obtain given answer from correct working |
|  |  |  |  |  | [5] |  |
| $\begin{gathered} \text { ALT } \\ 2 \\ 5(\mathrm{a}) \end{gathered}$ | Using $y$-axis$r^{3} \int^{\frac{\pi}{2}} \sin ^{3} \theta$ |  |  |  | M1 | Use of correct integral. Limits not needed here. <br> Accept an integral of the form: $k r^{3} \int \sin ^{3} \theta \mathrm{~d} \theta$ |
|  | $\begin{aligned} & r^{3} \int_{0}^{\frac{\pi}{2}}\left(1-\cos ^{2} \theta\right) \sin \theta \mathrm{d} \theta \\ & \quad=r^{3}\left[-\cos \theta+\frac{\cos ^{3} \theta}{3}\right]_{0}^{\frac{\pi}{2}} \end{aligned}$ |  |  |  | A1 | Correct integration, ignore limits. Correct expression. |
|  | $=\frac{2}{3} r^{3}$ |  |  |  | A1 | Correct use of limits |
|  | $\bar{x}=\frac{\frac{2}{3} r^{3}}{\frac{1}{2} \pi r^{2}}$ |  |  |  | M1 | Complete method to obtain distance. <br> Use of correct formula. $\rho$ must appear on both sides or neither. |
|  | $=\frac{4 r}{3 \pi} *$ |  |  |  | A1* | Obtain given answer from correct working |
|  |  |  |  |  | [5] |  |
| 5b |  | large | Small removed | Small added |  |  |
|  | mass | $8 \pi a^{2}$ | $2 \pi a^{2}$ | $2 \pi a^{2}$ | B1 | Correct mass ratios |
|  | $\begin{aligned} & \text { From } \\ & A C \end{aligned}$ | $\frac{16 a}{3 \pi}$ | $\frac{8 a}{3 \pi}$ | (-) $\frac{8 a}{3 \pi}$ | B1 | Correct distances |
|  | Moments about $A C$ |  |  |  | M1 | All terms required. Dimensionally correct or equivalent for a parallel axis. Condone sign errors. If column vectors are used, this mark is awarded once the equation is written separate to the column vectors. |


|  | $8 \pi a^{2} \times \frac{16 a}{3 \pi}-2 \pi a^{2} \times \frac{8 a}{3 \pi}-2 \pi a^{2} \times \frac{8 a}{3 \pi}$ <br> $=8 \pi a^{2} d$ | A1 | Correct unsimplified equation. |
| :--- | :--- | :--- | :--- |
|  | $\frac{96 a}{3 \pi}=8 d \Rightarrow d=\frac{4 a}{\pi} *$ | A1* | Obtain given value from <br> correct working. Need to see at <br> least some simplification. |
| 5c | Moments about perpendicular axis through <br> $A$ | M1 | Dimensionally correct. <br> Need all terms. <br> Or equivalent for a parallel <br> axis |
|  | From $A$ <br> $4 a \times 8 \pi a^{2}-2 a \times 2 \pi a^{2}+6 a \times 2 \pi a^{2}=8 \pi a^{2} \bar{x}$ | A1ft | Ansimplified equation with at <br> most one error. <br> Correct unsimplified equation <br> Follow their mass ratio |
| $\Rightarrow \bar{x}=5 a$ | A1 | Correct only. If measured from <br> $B$, distance is $a$ |  |
|  | Correct use of trig to find an expression for <br> $\tan \theta$ | M1 | tan $\theta=\frac{d}{\bar{x}}$ or tan $\theta=\frac{\bar{x}}{d}$ where <br> $\bar{x}$ is distance from $A$. |
|  | $\tan \theta=\frac{4}{5 \pi}$ | A1 | Only |
|  | $[6]$ | $(16)$ |  |
|  |  |  |  |


| 6a | In equilibrium | M1 | Need all three forces. Dimensionally correct |
| :---: | :---: | :---: | :---: |
|  | $m g+4 m g \frac{l-e}{l}=4 m g \frac{e}{l}$ | A1 A1 | Unsimplified equation with at most one error Correct unsimplified equation |
|  | $\begin{aligned} & 5 l=8 e \Rightarrow e=\frac{5 l}{8}, \\ & A E=l+\frac{5 l}{8}=\frac{13 l}{8} \end{aligned}$ | A1* | Obtain given answer from correct working. <br> Must see $A E=$ |
| ALT1 | $m g+4 m g \frac{(2 l-A E)}{l}=4 m g \frac{(A E-l)}{l}$ | M1 <br> A1 A1 | Need all three forces. <br> Dimensionally correct <br> Unsimplified equation with at most one error <br> Correct unsimplified equation |
|  | $A E=\frac{13 l}{8} \quad *$ | A1* | Obtain given answer from correct working. Must see $A E$ = |
| ALT2 | $m g+4 m g \frac{\left(\frac{l}{2}-e\right)}{l}=4 m g \frac{\left(\frac{l}{2}+e\right)}{l}$ | M1 <br> A1 <br> A1 | Need all three forces. <br> Dimensionally correct <br> Unsimplified equation with at most one error Correct unsimplified equation |
|  | $e=\frac{l}{8}, \quad A E=l+\frac{l}{2}+\frac{l}{8}=\frac{13 l}{8} \quad *$ | A1* | Obtain given answer from correct working |
|  |  | [4] |  |
| 6b | Equation of motion | M1 | Need all terms. Dimensionally correct. Condone use of $a$ for acceleration. |
|  | $4 m g \frac{\frac{5 l}{8}+x}{l}-4 m g \frac{\frac{3 l}{8}-x}{l}-m g=-m \ddot{x}$ | $\begin{aligned} & \mathrm{A} 1 \\ & \mathrm{~A} 1 \end{aligned}$ | Unsimplified equation with at most one error. <br> Correct unsimplified equation Note: the question states $x$ is measured vertically down. |
|  | $\Rightarrow-m \ddot{x}=\frac{8 m g}{l} x, \ddot{x}=-\frac{8 g}{l} x *$ | A1* | Obtain given answer from correct working. Must use $\ddot{x}$ |
|  |  | [4] |  |
| 6c | Use of $v^{2}=\omega^{2}\left(a^{2}-x^{2}\right)$ with $a=\frac{3 l}{8}$ | M1 | Or use of equivalent correct formula |
|  | $=\frac{8 g}{l}\left(\frac{9}{64} l^{2}-\frac{1}{64} l^{2}\right)$ | A1 | Correct unsimplified expression for $v$ or $v^{2}$ |
|  | $v=\sqrt{g l}$ | A1 | Correct only |
|  |  | [3] |  |


| 6d | $x=\frac{3 l}{8} \cos \omega t$ | B1 | Use of relevant formula with correct amplitude $x=\frac{3 l}{8} \cos \omega t$ or $x=\frac{3 l}{8} \sin \omega t$ |
| :---: | :---: | :---: | :---: |
|  | Use of $-\frac{l}{8}=\frac{3 l}{8} \cos \omega t$ <br> or <br> $\frac{l}{8}=\frac{3 l}{8} \sin \omega t$ and correct use of $\frac{1}{2} \times \frac{2 \pi}{\omega}$ <br> or <br> $\frac{l}{8}=\frac{3 l}{8} \cos \omega t$ and correct use of $\pi-\cos ^{-1}\left(\frac{1}{3}\right)$ | M1 | Complete method to find $t$ or required time $t=\frac{1}{\omega} \cos ^{-1}\left(-\frac{1}{3}\right)$ <br> or <br> $t=\frac{1}{\omega} \sin ^{-1}\left(\frac{1}{3}\right)$ with $\frac{1}{2}$ period <br> or <br> $t=\frac{1}{\omega} \cos ^{-1}\left(\frac{1}{3}\right)$ with $\pi$ |
|  | Required time $\frac{2}{\omega} \cos ^{-1}\left(\frac{-1}{3}\right)=\sqrt{\frac{l}{2 g}} \cos ^{-1}\left(\frac{-1}{3}\right)$ <br> or $\frac{\pi}{\omega}+\frac{2}{\omega} \sin ^{-1}\left(\frac{1}{3}\right)=\sqrt{\frac{l}{8 g}}\left(\pi+2 \sin ^{-1}\left(\frac{1}{3}\right)\right)$ <br> or $\frac{2}{\omega}\left[\pi-\cos ^{-1}\left(\frac{1}{3}\right)\right]=\sqrt{\frac{l}{2 g}}\left[\pi-\cos ^{-1}\left(\frac{1}{3}\right)\right]$ | A1 | Or equivalent, accept $\begin{aligned} & 1.91 \sqrt{\frac{l}{2 g}}, 1.35 \sqrt{\frac{l}{g}}, 0.43 \sqrt{l} \\ & 3.82 \sqrt{\frac{l}{8 g}} \\ & \cos ^{-1}\left(\frac{-1}{3}\right)=1.91 \ldots \end{aligned}$ |
|  |  | [3] |  |
|  |  | (14) |  |


| 7a | Conservation of mechanical energy: | M1 | All terms required. Dimensionally correct $\cos \theta=\frac{5}{13}, \sin \theta=\frac{12}{13}$ |
| :---: | :---: | :---: | :---: |
|  | $\frac{1}{2} m u^{2}=\frac{1}{2} m v^{2}+m g(r+r \cos \theta)$ | A1 | Correct unsimplified equation |
|  | $v^{2}=u^{2}-\frac{36}{13} g r \quad *$ | A1* | Obtain given answer from correct working |
|  |  | [3] |  |
| 7b | Equation of motion | M1 | All terms required. <br> Dimensionally correct. Condone sign errors and sin/cos confusion. <br> Condone use of $R=0$ |
|  | $R+m g \cos \theta=\frac{m v^{2}}{r}$ | A1 | Correct unsimplified equation. Condone (strict) inequality the right way round. |
|  | Use $R \ldots .0$ and solve for $u^{2}$ | M1 | Complete method to obtain $u^{2}$ Condone use of $R=0$ or $R>0$ |
|  | $\begin{aligned} & \frac{m v^{2}}{r}-m g \cos \theta \ldots 0 \\ & \Rightarrow u^{2}-\frac{36}{13} g r \ldots \frac{5}{13} g r, \quad u^{2} \ldots \frac{41}{13} g r \end{aligned}$ | A1* | Obtain given answer from correct working. <br> Must have stated the inequality $R \geq 0$ <br> If there is no reference to $R$, the max mark in (b) is M1A1M1A0* |
|  |  | [4] |  |
| 7c | $B C=2 r \sin \theta=\frac{24}{13} r$ | B1 | Or equivalent $B C=1.846 \ldots r$ |
|  | Relevant vertical motion Eg time to return to the level of $B C$ | M1 | Complete method vertically using suvat |
|  | $t=\frac{2 v \sin \theta}{g}=\frac{24 v}{13 g}$ | A1 | Correct unsimplified expression for time <br> Accept $\frac{24}{13 g} \times 4 \sqrt{\frac{g r}{13}}, \frac{24}{13} \sqrt{\frac{16 r}{13 g}}$ $\frac{96}{13} \sqrt{\frac{r}{13 g}}, 0.65 \sqrt{r}$ |
|  | Relevant horizontal motion Eg distance travelled by $P$ | M1 | Complete method horizontally |
|  | $=(v \cos \theta) t=v^{2} \times \frac{120}{169 g}$ | A1 | Correct unsimplified expression for distance $0.87 r, \frac{1920}{2197} r, 0.0892 \mathrm{gr}$ |
|  | $=\frac{16 g r}{13} \times \frac{120}{169 g}=\frac{160 r}{169} \times \frac{12}{13}<2 r \times \frac{12}{13}$ | A1* | Obtain given conclusion from correct working |


|  | hence falls into the bowl * |  |  |
| :---: | :---: | :---: | :---: |
| ALT 1 for last 3 marks | Horizontal: time, $T$, required to travel the length $B C$ | M1 | Complete method horizontally |
|  | $\begin{aligned} & 2 r \sin \theta=v \cos \theta \times T \\ & T=\frac{2 r \frac{12}{13}}{4 \sqrt{\frac{g r}{13}} \times \frac{5}{13}}=1.38 \sqrt{r} \end{aligned}$ | A1 | Correct unsimplified expression for $T$ |
|  | $t<T \text { since } 0.654 \sqrt{r}<1.38 \sqrt{r}$ <br> hence falls into the bowl * | A1* | Obtain given conclusion from correct working |
| ALT 2 for last 3 marks | Horizontal: speed, $V$, required to reach C | M1 | Complete method horizontally |
|  | $\begin{aligned} & -V \sin \theta=V \sin \theta-g \frac{2 r \sin \theta}{V \cos \theta} \\ & \Rightarrow V=\sqrt{\frac{g r}{\cos \theta}}=\sqrt{\frac{13 g r}{5}} \end{aligned}$ | A1 | Correct unsimplified expression for $V$ |
|  | $v<V \text { since } \sqrt{\frac{13 g r}{5}}<\sqrt{\frac{16 g r}{13}}$ <br> hence falls into the bowl * | A1* | Obtain given conclusion from correct working |
|  | SC: If range formula is quoted correctly award M1A1M1A1. $\text { Range }=\frac{2 v^{2} \sin \theta \cos \theta}{g}$ |  |  |
|  |  | [6] |  |
|  |  | (13) |  |

